*Roll No.* ..............................

# 21183

## B. Sc. (Pass Course) 2nd Semester Examination – May, 2019

### MATHEMATICS (VECTOR CALCULUS)

**Paper : 12BSM123**

*Time : Three hours ]* *[ Maximum Marks : 40*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

***Note :*** Attempt ***five*** questions in all, selecting ***one*** question from each Section. Section – **V** is ***compulsory***.

### SECTION – I

**1.** (a) To show that : $3\frac{1}{2}$

$$\vec{a}'.\left(\vec{b}' \times \vec{c}'\right) = \frac{1}{\vec{a}.(\vec{b} \times \vec{c})}$$

P. T. O.











(b) If $\vec{f}$ is a differentiable vector function of scalar variable $t$ and $\left(\vec{f}\right) = f$ then prove $\frac{d}{dt}\left(\vec{f}^2\right) = 2f\frac{df}{dt}$ and $\vec{f}.\frac{d\vec{f}}{dt} = f\frac{df}{dt}$. $3\frac{1}{2}$

**2.** (a) If $\vec{r}$ is a vector of a scalar $t$ and $\left|\vec{r}\right| = r$ and $\vec{a}$ is constant vector then differentiate w. r. t. $t$ $\frac{\vec{r}+\vec{a}}{\vec{r}^2+\vec{a}^2}$ $3\frac{1}{2}$

(b) Find the unit tangent vector to the curve $\vec{r} = a(t-\sin t)\hat{i} + a(1-\cos t)\hat{j}$ at any point $t$. $3\frac{1}{2}$

### SECTION – II

**3.** (a) Show that $\nabla f(r) \times \vec{r} = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. $3\frac{1}{2}$

(b) If $\vec{f}$ is vector point function and $\phi$ is scalar point function having continuous second order partial derivative then prove $\nabla \times (\phi\vec{f}) = (\nabla\phi) \times \vec{f} + \phi(\nabla \times \vec{f})$. $3\frac{1}{2}$

**4.** (a) Prove that curve $\left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}\left(\vec{a}.\vec{r}\right)$

Where $\vec{a}$ is constant vector. $3\frac{1}{2}$

(b) Show that the function $\frac{1}{r}$, $|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$ is harmonic fu

provided $r \neq 0$.

## SECTION – III

**5.** (a) If $u, v, w$ are orthogonal curvilinear co-ord then $\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial v}, \frac{\partial \vec{r}}{\partial w}$ and $\nabla u$, $\nabla v$, $\nabla w$

reciprocal system of vectors.

(b) To show that $\nabla . \vec{f} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3\right.$

$\left.\frac{\partial}{\partial v}(h_3 h_1 f_2) + \frac{\partial}{\partial w}(h_1 h_2 f_3)\right]$ where $\vec{f} = f_1 \hat{e}_1$

$+ f_3 \hat{e}_3$.

**6.** (a) If (r, θ, ϕ) are spherical Co-ordinates shov $\nabla \phi = \nabla \times (r \operatorname{cosec} 0 \Lambda \theta)$.

(b) Express the velocity and acceleration of a p in spherical co-ordinates.

## SECTION – IV

**7.** (a) Find the circulation of $\vec{f}$ around the cur where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the $x^2 + y^2 = 1, z = 0$.













(b) Calculate $\int_C \left[(x^2+y^2)\hat{i} + (x^2-y^2)\hat{j}\right] d\vec{r}$. where C is the curve $y^2 = x$ joining (0,0) to (1,1). $3\frac{1}{2}$

**8.** (a) State and prove Green's theorem. $3\frac{1}{2}$

(b) Verify Stoke's theorem for the function $\vec{f} = x^2\hat{i} + xy\hat{j}$ integrated around the square in the plane $z = 0$ whose sides are along the lines $x = 0, x = a, y = 0, y = a$. $3\frac{1}{2}$

## SECTION – V

(a) Define divergence of a vector function. 2

(b) Define Laplacian operator. 2

(c) Show that div. (curl $\vec{f}$) = 0 2

(d) State Stokes theorem. 2

(e) If $\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j} + t\hat{k}$ find $\left|\frac{d\vec{r}}{dt}\right|$. 2

(f) Prove that div. $\vec{r} = 3$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 2