

SECTION – D

8. (a) Find the volume of a spherical cap of height  $h$  cut off from a sphere of radius  $a$ .
- (b) Find, by double integration, the volume generated by revolving the cardioid  $r = a(1 + \cos\theta)$  about the initial line.

9. (a) Evaluate  $\iint r^3 dr d\theta$ , over the area bounded between the circles  $r = 2 \cos \theta$  and  $r = 4 \cos \theta$ .
- (b) By changing the order of integration, evaluate

$$\int_0^{\infty} \int_0^x x e^{-\frac{x}{y}} dy dx.$$

Roll No. ....

24002

B. Tech 1st Semester (Common for All Branches) Examination – December, 2017

MATHEMATICS-I

Paper : Math-101-F

Time : Three Hours ] [ Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt total five questions, selecting one question from each Sections. Question No. 1 is compulsory. All questions carry equal marks.

1. (a) Test the convergence of :

$$\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n} + \sqrt{n+1}} \right).$$

- (b) For what values of  $\lambda$  and  $\mu$  do the equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$  have unique solution.

(c) If  $x = r \cos \theta, y = r \sin \theta$  find  $\frac{\partial^2 r}{\partial x^2}$  and  $\frac{\partial^2 \theta}{\partial y^2}$ .

(d) Define Beta and Gamma functions. Also give relationship between them.

**SECTION - A**

2. (a) Discuss the convergence of the series :

$$\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \frac{x^4}{5\sqrt{6}} + \dots \infty.$$

(b) Test the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{n!}{(n+1)^n} x^n$$

3. Test  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$  for convergence and absolute convergence.

**SECTION - B**

4. (a) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

by reducing it in its normal form.

(b) Are the following vectors linearly dependent ? If so, find the relation between them :

$$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (4,5,6), x_4 = (1,8,-3).$$

5. Diagonalise the matrix :

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \text{ and hence find } A^4.$$

**SECTION - C**

6. (a) Using Taylor's series, expand  $\sin x$  in powers of  $\left(x - \frac{\pi}{2}\right)$ . Hence find the value of  $\sin 91^\circ$  correct to four decimal places.

(b) Find the radius of curvature for the curve  $r = a(1 + \cos \theta)$ .

7. (a) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere  $x^2 + y^2 + z^2 = 1$ .

(b) Evaluate the integral :

$$\int_0^{\pi/2} \frac{\log(1 + a \sin^2 x)}{\sin^2 x} dx.$$