Roll No. ....

1982

B. E. / B. Tech. 1st Semester Examination – December, 2013

MATHEMATICS-I

'E' Scheme

Paper : MATH-101(E)

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, taking at least *two* questions from each Part. All questions carry equal marks.

PART – A

1. (a) Discuss the convergence of the series :

$$\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{5}\right)^3 x^3+.....\infty$$

(b) Discuss the convergence of the series :

$$\frac{1}{6}x^2-\frac{2}{11}x^3+\frac{3}{16}x^4-\frac{4}{21}x^5+\frac{5}{26}x^6........$$

2. (a) Test the convergence of the series :

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....(-1)^{n-1}\frac{x^n}{n}+.....\infty$$

(b) Discuss the maxima and minima of :

$$\sin x+\sin y+\sin(x+y)$$

3. (a) Expand sin x in powers of $\left(x-\frac{\pi}{2}\right)$. Hence find the value of sin 91°.

(b) Show that for the parabola $y^2=4ax$, ρ^2 varies as $(SP)^3$, where ρ is the radius of curvature at any point P of the parabola and S is the focus of the parabola.

4. (a) If $x+y=2e^{\theta}\cos\phi$, show that :

$$\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\phi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$$

(b) If $u=\operatorname{cosec}^{-1}\left(\frac{(x)^{\frac{1}{2}}+(y)^{\frac{1}{2}}}{(x)^{\frac{1}{3}}+(y)^{\frac{1}{3}}}\right)^{\frac{1}{2}}$, then prove that :

$$x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+2xy\frac{\partial^2 u}{\partial x\partial y}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$$

PART – B

5. (a) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ and hence evaluate the same.

(b) Show by double integration, that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

6. (a) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

(b) Prove that :

$$\Gamma m \, \Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$$

7. (a) Find the directional derivatives of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

(b) Define curl of a vector point function and give its geometrical interpretation, also prove that curl curl F = grad (div F) – ∇^2F.

8. (a) Verify divergence theorem for $\vec{F} = 4xi - 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

(b) Apply Green's theorem to evaluate :

$$\oint_C \left[\left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy\right]$$

where C is the boundary of the region defined by the $x = 0$, $y = 0$, $x + y = 1$.