(b) Prove that the composition of two linear transformation is again a linear transformation. $7\frac{1}{2}$

UNIT – IV

8. (a) Find the eigen value and eigen vectors for the matrix $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. 8

(b) Show that if matrix A is orthogonal then A' and A^{-1} are also orthogonal further, show that the determinant of orthogonal matrix is ± 1. 7

9. (a) Diagonalize the matrix $A=\begin{bmatrix}-1 & 3\\ -2 & 4\end{bmatrix}$. 7

(b) Using Gram-Schmidt orthogonalization process find an orthonormal basis of $V_3(R)$ with standard inner product defined on it, given the basis {(1,0,1), (1,0,–1), (0,3,4)}. 8

Roll No. ....

3008

B. Tech. 1st Semester (CSE)

Examination – December, 2018

MATHEMATICS - I (CALCULUS & LINEAR ALGEBRA)

Paper : BSC-Math-103-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting at least ***one*** question from each Unit. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Verify Rolle's theorem for $f(x)=(x+2)^3\,(x-3)^4$ in (–2, 3).

(b) Show that the vectors (1, 1, 2, 4), (2, –1, –5, 2), (1, –1, –4, 0) and (2, 1, 1, 6) are linearly dependent.

(c) If D = diag $[d_1, d_2, d_3]$, $d_1, d_2, d_3 \neq 0$. Prove that $D^{-1}=\left[d_1^{-1}, d_2^{-1}, d_3^{-1}\right]$

(d) Define basis and dimension of a vector space.

(e) Find the matrix representing the transformation $T : R^2 \to R^3$ given by $T(x, y) = (3x - y, 2x + 4y, 5x - 6y)$ relative to standard basis of R^2 and R^3.

(f) Define an Inner product space. 15

UNIT – I

2. (a) Evaluate $\underset{x \to 0}{Lt} (\cot x)^{1/\log x}$. $7\frac{1}{2}$

(b) Show that the equation of the evolutes of the parabola $x^2 = 4ay$ is $4(y - 2a)^3 = 27ax^2$. $7\frac{1}{2}$

3. (a) Find the surface of the solid generated by the revolution of the curve $x = a\cos^3 t, y = a\sin^3 t$ about the $x - axis$. $7\frac{1}{2}$

(b) Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. $7\frac{1}{2}$

UNIT – II

4. (a) If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, find AB or BA, whichever exists. $7\frac{1}{2}$

(b) Determine the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. $7\frac{1}{2}$

5. (a) Solve the equation $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$ by using Cramer's role. $7\frac{1}{2}$

(b) Use Gaus elimination to solve the system of linear equation : $7\frac{1}{2}$

$$2x_2 + x_3 = -8; x_1 - 2x_2 - 3x_3 = 0; -x_1 + x_2 + 2x_3 = 3$$

UNIT – III

6. (a) Let $T : U \to V$ be a linear transformation Then show that range of T i.e. R (T) is a subspace of V. and null space of T i.e. N (T) is a subspace of U. 7

(b) For the linear transformation $T : R^2 \to R^3$ s. t. $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$. Verity that Rank (T) +Nullify (T) = dim R^2. 8

7. (a) Show that the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$ is invertible and find T^{-1}. $7\frac{1}{2}$