24018

B.Tech. 2nd Semester F-Scheme Examination, May–2018

MATHEMATICS–II

Paper–MATH–102–F

(Common for All Branches)

Time allowed : 3 hours] *[Maximum marks : 100*

Note : Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.

1. (a) If $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$. Find Div $\vec{F}$ and curl $\vec{F}$

(b) Show that, if R = sin ωt + B cos ωt, where A, B, ω are constants, then

$$\frac{d^2R}{dt^2} = -\omega^2 R \text{ and } R \times \frac{dR}{dt} = -\omega \times R$$

(c) Solve $pq = p + q$.

(d) Find Laplace transformation of $\frac{e^{-t}\sin t}{t}$

(e) Solve $\frac{\partial^2 z}{\partial y\,\partial x} = xy$

(f) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-2t}$

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(g) Find the inverse Laplace transformation of $\frac{s+3}{s^2+6s+13}$

(h) Solve $(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$.

Unit–I

2. (a) Give the geometrical interpretation of gradient, also prove that curl curl $\vec{F}$ = grad (div $\vec{F}$) $- \nabla^2\vec{F}$.

(b) Prove that

(i) $\nabla \times (\vec{F} \times \vec{G}) = (\nabla.\vec{G})\vec{F} - (\nabla.\vec{F})\vec{G} + (\vec{G}.\nabla)\vec{F} - (\vec{F}.\nabla)\vec{G}$

(ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

3. (a) Verify Green's theorem in the plane for

$\oint_C [(x^2 + xy)\,dx + (x^2 + y^2)\,dy]$, where C is the square formed by the lines $y = \pm 1, x = \pm 1$.

(b) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\,i - yz^2\,j - y^2 z\,k$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy-plane.

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Unit–II

4. (a) Solve the following differential equation :

$$(2x^2y - 3y^4)\,dx + (3x^3 + 2xy^3)\,dy = 0$$

(b) A constant e.m.f. E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. If the initial current is zero, show that the current builds up to half its theoretical maximum is (L log 2)/R seconds.

5. (a) By the method of variation of parameters :

solve $\frac{d^2y}{dx^2} + 4y = \tan 2x.$

(b) Solve the equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x).$$

Unit–III

6. (a) Find the Laplace transform of

(i) coshat. cosat

(ii) $\int_0^t t \sin 3t\, dt$

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(b) Find inverse Laplace transformation of $(se^{-s/2} + \pi e^{-s}) / (s^2 + \pi^2)$.

7. (a) Using Convolution theorem, find Inverse Laplace transformation of $\frac{s}{(s^2+a^2)^2}$

(b) Solve

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 5x = e^{-t}\sin t,\ x(0) = 1,\ x'(0) = -1,$$

using Laplace transform.

Unit–IV

8. (a) Solve the following differential equation

$$(y+z)\,p - (z+x)\,q = (x-y)$$

(b) Solve the equation by Charpit's method

$$(p^2 + q^2)\,y = qz$$

9. (a) Find the differential equation of all planes which are at a constant distance 'a' from the origin.

(b) Using method of separation of variables,

solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

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