Roll No. ....

24022

B. Tech. 3rd Semester (CSE)

Examination – December, 2018

MATHEMATICS - III

Paper : Math-201-F

Time : Three Hours]   [Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, by selecting *one* Question from each Section. Question No. 1 is *compulsory.*

1. *Compulsory* question :

   (a) State Dirichlet's condition for existence of fourier series of a function.

   (b) If $F_s(S)$ and $F_c(S)$ are fourier sine and cosine transform of $f(x)$ respectively, then show that :

   (i) $F_s(xf(x)) = \dfrac{-d}{dx}\{F_c(s)\}$

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(ii) $F_c(xf(x)) = \dfrac{d}{ds}\{F_s(s)\}$

(c) Show that :

$$f(z) = \frac{x^2 y^3 (x+iy)}{x^6 + y^{10}}, \; z \neq 0, \; f(0) = 0$$

is not analytic at origin.

(d) Evaluate :

$$\int_0^{1+i} (x^2 - iy)\, dz \text{ along the path } y = x^2$$

(e) Find the radius of convergence of the power series : http://haryanapapers.com

$$\sum \frac{2^{-n} z^n}{1 + in^2}$$

(f) A variate X has the probability distribution :

X :	–3	6	9
P[X = x] :	1/6	1/2	1/3

Find E $\left[(2x+1)^2\right]$.

(b) Using Dual simplex method, solve :   10

Min $z = x_1 + 2x_2 + 3x_3$

Subject to :

$2x_1 - x_2 + x_3 \geq 4,\; x_1 + x_2 + 2x_3 \leq 8,\; x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$.

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(b) A certain screw making machine produces on average 2 defective screw out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws.　　10

SECTION – D

8. (a) 325 men out of 600 men chosen from a big city were found to be smokers. Does this information support the conclusion that the majority of men in the city are smokers.　　10

(b) Two independent samples of size 7 and 9 have the following values:　　10

Sample A :	10	12	10	13	14	11	10	–	–
Sample B	10	13	15	12	10	14	11	12	11

Test whether the difference between the mean is significant.

9. (a) Using Simplex method, solve :　　10

Max $z = x_1 + x_2 + 3x_3$

Subject to $3x_1 + 2x_2 + x_3 \leq 3, 2x_1 + x_2 + 2x_3 \leq 2,$
$x_1, x_2, x_3 \geq 0.$

(g) Write a short note on :

(i) Test of significance

(ii) Errors

(h) Solve graphically Max $z = 3x + 4y$, subject to the constraints $2x + 4y \leq 40; 2x + 5y \leq 180, x \geq 0, y \geq 0.$

　　20

SECTION – A

2. (a) Find the fourier series expansion of $f(x)$ if :　　10

$$f(x) = \begin{cases} -\pi & , & -\pi < x < 0 \\ x & & 0 < x < \pi \end{cases}$$

and hence deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ... = \dfrac{\pi^2}{8}$.

(b) Develop $f(x)$ in a fourier series in the interval $(0,2)$ if. :　　10

$$f(x) = x, 0 < x < 1$$

$$= 0, 1 < x < 2$$

3. (a) Express $f(x) = \begin{cases} 1 & , \quad 0 \le x \le \pi \\ 0 & \quad x > \pi \end{cases}$ as a fourier sine integral and hence evaluate

$$\int_0^\infty \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda)\, d\lambda .$$ 10

(b) Find the fourier cosine transform of e^{-x^2}. 10

SECTION – B

4. (a) Express Log (log i) in the form $A + iB$. 10

(b) If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$, show that : 10

$$e^{2\phi} = \pm\cot\alpha/2 \text{ and } 2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha$$ 10

5. (a) If $w = \phi + i\psi$ represents the complex potential function for an electric field and $\psi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, determine ϕ. 10

(b) Evaluate $\oint_C \dfrac{e^{-2z}}{(z+1)^3}\, dz$, where C is the circle $|z| = 2$ by using Cauchy's integral formula. 10

SECTION – C

6. (a) Expand $\dfrac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$ in Laurent series. 10

(b) Evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{1 - 2r\sin\theta + r^2}, (0 < r < 1).$$ 10

7. (a) A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y. 10